TIMETABLING IN TRANSPORTATION

Introduction

- In transportation industry planning and scheduling problems abound.
- Variety is due to many models of transportation, e.g., shipping, airlines and railroads.
- The equipment and resources involved:
  - ships and ports,
  - planes and airports,
  - trains, tracks and railway stations,
- have different characteristics, costs and planning horizons.

Tanker scheduling

- Companies have usually *company-owned* and *chartered* ships.
- Objective is typically to minimize the total cost of transporting all cargoes.
  - n is the number of cargoes to be transported
  - T is the number of company-owned tanks
  - p is the number of ports
- S_i is the set of all possible schedules for ship i

Tanker scheduling

- Schedule l for ship i, $l \in S_i$, is represented by:

$$\begin{matrix} a_{i1}^l \\ a_{i2}^l \\ \vdots \\ a_{in}^l \end{matrix}$$

- a_{ij}^l is 1 if under schedule l ship i transports cargo j
- c_i^l is incremental cost of operating a company-owned ship i versus keeping it idle over planning horizon

Tanker scheduling

- c_j^* is the amount to be paid to transport cargo j on a chartered ship.
- Amount of money saved by operating ship i according to schedule l:

$$\pi_i^l = \sum_{j=1}^{n} a_{ij}^l c_j^* - c_i^l$$

- Decision variable x_i^l is 1 if ship i follows schedule l.

IP Tanker Scheduling Problem

$$\text{maximize} \quad \sum_{i=1}^{T} \sum_{l \in S_i} \pi_i^l x_i^l$$

subject to

$$\sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x_i^l \leq 1 \quad j = 1, \ldots, n$$

Each cargo can be assigned to at most one tanker

$$\sum_{l \in S_i}^{T} x_i^l \leq 1 \quad i = 1, \ldots, T$$

Each tanker can be assigned to at most one schedule

$$x_i^l \in \{0,1\} \quad l \in S_i, \ i = 1, \ldots, T$$

Tanker Scheduling Problem

- This problem is known as the **set-packing problem**.
- It is solved by a branch-and-bound algorithm.
- First, a collection of candidate schedules must be generated for each ship in the fleet.
- Upper and lower bounds must be defined.

Example 11.2.1

- Three ships and 12 cargoes. 15 feasible schedules:

Cargoes	a_{1j}^1	a_{1j}^2	a_{1j}^3	a_{1j}^4	a_{1j}^5	a_{2j}^1	a_{2j}^2	a_{2j}^3	a_{2j}^4	a_{2j}^5	a_{3j}^1	a_{3j}^2	a_{3j}^3	a_{3j}^4	a_{3j}^5
1	1	0	0	1	1	0	1	0	0	0	0	0	0	1	0
2	1	0	0	0	0	1	0	0	0	0	0	1	0	1	1
3	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0
4	0	1	1	1	0	1	0	1	0	0	0	0	0	0	0
5	1	1	0	0	0	0	0	0	1	0	0	0	1	0	1
6	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0
7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
8	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0
9	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0
10	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0
11	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0
12	0	0	0	1	0	0	0	0	0	0	1	0	1	1	1
	Ship 1					Ship 2					Ship 3				

Problem data

- If cargo is transported by a charter, cost is:

1	2	3	4	5	6	7	8	9	10	11	12
1429	1323	1208	512	2173	2217	1775	1885	2468	1928	1634	741

- Operating costs of tankers for each schedule:

Schedule l	1	2	3	4	5
tanker 1	5658	5033	2722	3505	3996
tanker 2	4019	6914	4693	7910	6868
tanker 3	5829	5588	8284	3338	4715

Formulation of IP problem

- Profits of each schedule can be computed:

Schedule l	1	2	3	4	5
tanker 1	-733	1465	1466	1394	858
tanker 2	1629	834	1113	-869	910
tanker 3	1525	1765	-1268	1789	1297

- Integer program:

maximize
$$\begin{aligned}&-773x_1^1+1465x_1^2+1466x_1^3+1394x_1^4+858x_1^5\\&+1629x_2^1+834x_2^2+1113x_2^3-869x_2^4+910x_2^5\\&+1525x_3^1+1765x_3^2-1268x_3^3+1789x_3^4+1297x_3^5\end{aligned}$$

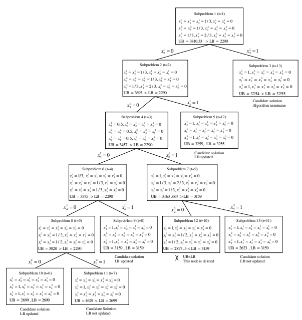
Formulation of IP problem

subject to
$$\begin{aligned}
&x_1^1+x_1^4+x_1^5+x_2^2+x_3^4\le 1\\
&x_1^1+x_2^1+x_3^2+x_3^4+x_3^5\le 1\\
&x_1^3+x_1^5+x_2^4+x_2^5\le 1\\
&x_1^2+x_1^3+x_1^4+x_2^1+x_2^3\le 1\\
&x_1^1+x_1^2+x_2^4+x_3^3+x_3^5\le 1\\
&x_1^4+x_1^5+x_2^2+x_2^5+x_3^1\le 1\\
&x_2^3+x_2^4+x_3^5\le 1\\
&x_1^2+x_2^1+x_2^3+x_2^4+x_2^5\le 1\\
&x_1^3+x_2^2+x_2^5+x_3^1+x_3^2+x_3^3\le 1\\
&x_1^2+x_2^1+x_3^1+x_3^2\le 1\\
&x_2^2+x_2^3+x_3^2+x_3^3+x_3^4\le 1\\
&x_1^4+x_3^1+x_3^3+x_3^4+x_3^5\le 1\\
&x_1^1+x_1^2+x_1^3+x_1^4+x_1^5\le 1\\
&x_2^1+x_2^2+x_2^3+x_2^4+x_2^5\le 1\\
&x_3^1+x_3^2+x_3^3+x_3^4+x_3^5\le 1\\
&x_i^l\in\{0,1\}
\end{aligned}$$

Initial upper bound

- Initial upper bound can be obtained by solving the linear relaxation of the IP, i.e., allowing x_i^l to assume values between 0 and 1.
  - The solution is $x_1^2=x_1^3=x_1^5=1/3$, $x_2^1=x_2^5=1/3$ $x_3^1=1/3$, $x_3^4=2/3$.
- The value of this solution (upper bound) is 3810.33.
- (See tree of branch-and-bound problem).
- Optimal solution: $x_1^3=1$, $x_3^4=1$. Ship 2 is idle and cargoes 5, 6, 7, 8 and 10 are transported by charters. Optimal value: 3255.

Branch and bound tree



Airline routing and scheduling

- Problem: construct a daily schedule for a heterogeneous aircraft fleet.
- Two parts:
  - determine sequence of flight legs (routing)
  - determining exact times of start and finish (scheduling)
- Costumer demands (profit) can be estimated from past experience and marketing research.
- Additional constraints: number of planes, restrictions at airports, required connection flights, etc.

Formulation of the problem

- L is the set of flight legs
- T is the number of different aircraft types
- m_i is the number of available aircrafts of type i.
- Total number of aircrafts is $\sum_{i=1}^{T} m_i$
- L_i is the set of flight legs that can be flown by an aircraft of type i.
- S_i is the set of feasible schedules for aircraft of type i. It includes the empty schedule (0).

Formulation of the problem

- π_{ij} is profit generated by covering flight leg j with aircraft of type i.
- For each schedule $l \in S_i$ the total anticipated profit is:

$$\pi_i^l = \sum_{j=1}^{n} \pi_{ij} a_{ij}^l$$

- where a_{ij}^l is 1 if schedule l covers leg j and 0 otherwise.
- If an aircraft is assigned to an empty schedule the profit is π_i^0, which can be positive or negative.

Formulation

- P is the set of airports.
- P_i is the subset that accommodate aircrafts of type i.
- o_{ip}^l is 1 if the origin of schedule l is airport p.
- d_{ip}^l is 1 if the final destination of schedule l is airport p.
- Decision variable x_i^l is 1 if schedule l is assigned to an aircraft of type i.
- Integer decision variable x_i^0 is the number of unused aircrafts of type i.

Daily Airline Routing and Scheduling

$$\text{maximize} \quad \sum_{i=1}^{T} \sum_{l \in S_i} \pi_i^l x_i^l$$

subject to

$$\sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x_i^l = 1 \quad j \in L$$

Each flight leg has to be covered exactly once.

$$\sum_{l \in S_i} x_i^l = m_i \quad i = 1, \ldots, T$$

Maximum number of aircrafts of each type that can be used

$$\sum_{l \in S_i} (d_{ip}^l - o_{ip}^l) x_i^l = 0 \quad i = 1, \ldots, T,\ p \in P_i$$

Flow conservation constraints at the end of the day at each airport for each aircraft type

$$x_i^l \in \{0,1\} \quad l \in S_i,\ i = 1, \ldots, T$$

Daily Airline Routing and Scheduling

- The problem is a **Set Partitioning Problem** with additional constraints.
- Algorithm to solve the problem is also a branch-and-bound algorithm, known as a branch-and-price algorithm.

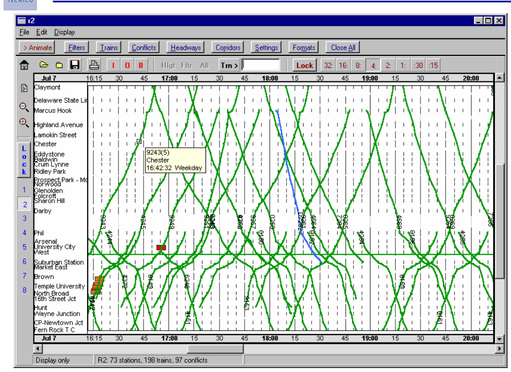
Example 11.3.1

- Two types of planes, $T = 2$. $m_1 = 2$, $m_2 = 2$.
- Twelve flights to be flown between 4 airports:
  - $p = 1$: San Francisco (SFO)
  - $p = 2$: Los Angeles (LAX)
  - $p = 3$: New York (NYC)
  - $p = 4$: Seattle (SEA)

Train timetabling

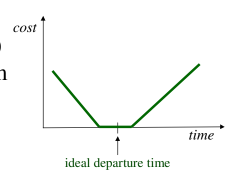
- Most common problem: single, one way track linking two major stations with smaller stations in between.
- Time in minutes: 1 to q (in one day $q = 1440$).
- Link j connects station $j - 1$ to j.
- There are L consecutive links and $L+1$ stations (0 to L)
- T is the set of trains, and T_j is the set intending to pass link j.
- A train can overtake another only at a station.

Time-distance graphic



Train timetabling

- Ideal timetable is determined by analyzing passenger behavior and preferences, but must satisfy track capacity constraints.
- There are preferred arrival and departure times at a station for each train.
- There is a cost (or revenue loss) associated with a deviation from the preferred arrival, stopping or departure times.



Train timetabling problem

- Formulated as a Mixed Integer Programming (MIP)
- For a single link the problem can be formulated as:
- **Objective**: minimize the cost of deviating from preferred arrival, stopping or departure times.
- Some **constraints**:
  - Least minimum time to traverse link j
  - Minimum amount of time stopping at a station i
  - Minimum headways between each link: there is a minimum amount of time between departures and arrivals at a station

Solution of train timetabling problem

- A railway system has a network of many links.
- Each single link can be solved by
  - branch-and-bound
  - a heuristic similar to the shifting bottleneck.
- **No optimality is guaranteed!**

➢ See Multi Model slides in the CD of Pinedo's book.

MULTI•MODAL applied systems